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An evolutionary learning ethical view

Masudul Alam Choudhury Institute of Islamic Banking and Finance, International Islamic University Malaysia, Kuala Lumpur, Malaysia, and M. Ishaq Bhatti

Department of Finance, La Trobe University, Melbourne, Australia

Abstract

Purpose – The purpose of this paper is to bring out the topic of ethics and economics in reference to the nature of complementarities that can exist between monetary and fiscal activities. The connector in such complementarities is the unity of knowledge that can be generated in the inter-causal relations between monetary and fiscal activities.

Design/methodology/approach – The methodology adopted is of measuring out by quantitative modeling how well there exists complementary relations or otherwise between the Central Bank and commercial bank in order to mathematically explain the role of participatory learning behavior using money, debt, and spending variables.

Findings – The argument placed takes the conceptual form of result to show that there would be a prolonged extension of the non-inflationary and technological induction of economic growth in a regime of complementing money and fiscal policies.

Originality/value – The role of the quantity of money in a non-inflationary economic growth is set against the background of the tripartite inter-causal relationships between the Central Bank, the commercial bank, and the real economy. Analytical methods used bring out the role of knowledge in the inter-causal relations termed as circular causation for the attainment of social well-being in response to a stable and advancing economy with the ethicality of unity of knowledge.

Keywords Causality, Creativity, Consciousness, Biocybernetics, Autopoiesis, Complexity **Paper type** Research paper

Background

In the heterodox field of ethico-economics and as it has been during the great depression of the 1930s the question on monetarism and fiscalism led into the burning theoretical issues of whether the depression of the time was caused by an inept equivalence between these two activities. Some argue (Friedman and Schwartz, 1963) that the Great Depression was caused by too little quantity of money in the economy. The state of expanded quantity of money, if that quantity could be absorbed by the real economy, would bring about productive effects by means of an equally expanded spending regime. The tripartite relationship of complementarities between the quantity of money, the spending regime, and the activation of the real economy also requires appropriate technological change. If such technological change further go in the direction of establishing and continuing life-sustaining regime of development



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then the complementarities between money, spending, and real economy can become the basis of price stabilization at non-inflationary real output and an expansionary potential to sustain long-run full-employment and real-output levels.

Review of the literature

The topic pointing out the need for integrating money and spending, particularized to monetarism and fiscalism, has been in the literature since sometime. Blaug comments are in place in this regard. Blaug (1993, p. 29) points out the contesting views held on economic stability and economic growth by the schools of monetarism and fiscalism: "The great debate between Keynesians and monetarists over the respective potency of spending and monetary policies has divided the economic profession, accumulating what is by now a simply enormous literature." Blaug (1993, p. 221) continues on to point out: "Monetarism never succeeded in clarifying the causal mechanism that produced its empirical results, sometimes even denying that these results required interpretation in the light of a supporting causal theory, and it failed to refute any but a crude travesty of the Keynesian theory it opposed. Keynesianism on the other hand, proved to be capable of absorbing monetarist ideas in a more sophisticated brand of macroeconomics that appears to be emerging from the fifteen-year-old melee."

In recent writing Desai (1989, pp. 146-150) points out the endogenous nature of money in the quantity theory of money and exchange in the following way: say the money aggregate M is related to three variables, x denoting price, y denoting output, and z denoting other variables. M is endogenous if the following circular relations hold and are both explainable as well as estimable:

$$M = f_1(x,y,z); \quad X = f_2(M,y,z); \quad Y = f_3(x,M,z); \quad Z = f_4(x,y,M)$$

These relations between the endogenous (circularly related by causation) variables would show strong endogeneity if the estimated relations are statistically significant, and thus lead to robust predictions of any one variable with respect to the other ones. Otherwise, the relationship is weakly endogenous, differentiated in reference to given levels of statistical significance. On the other hand, if one of the relationships is violated or is not estimable then the system is exogenous and this independence between the given variables also causes independence of the monetary aggregate from the other variables in the system of relationships. Such a referred to other variable along with money can be selected as fiscal variable. Thereby there can be endogenous relationship between these variables as explained above or the relationship can be of an exogenous type between money and fiscal variable. These developments in macroeconomic theory of monetary and fiscal dynamics are explained in Wikipedia (2011).

In fact the quantity theory of money and prices always treats money endogenously in relation to market spending. Milton Friedman is thus thought of recasting the monetary function in terms of market spending and thereby introducing money as an endogenous aggregate of the macroeconomic genre into microeconomic market function. Irving Fisher's version of the quantity theory equation explains the endogenous relationship between money and real economy by the medium of market spending. The Austrian school thought along the same lines of Fisher's version of quantity theory of money.

Choudhury et al. (2013) has contributed to the study of endogenous relationship between money and the real economy of market spending. The idea here too is to relate

the money and real economy relationship through the quantity theory equation of exchange. The endogenous theory of money is thereby established. For variations in such a theory of endogenous money, see Choudhury (1997).

Certain kinds of endogenous inter-causal relationship between money and spending were studied in the extended form of the quantity theory equation of exchange (Friedman, 1960, 1989). Such a systemic inter-causal relationship defines the complementary dynamics between the sectors, entities, and variables in question. Its earliest trace can be read off the classical quantity theory of money (Friedman, 1989).

Mishkin (2007) writes importantly on this issue. Mishkin's theory offers weak treatment of the above-mentioned problem caused by non-complementarities between monetary and spending dynamics, however. The result ensuing from Mishkin's monetary policy strategy on non-inflationary economic expansion is also one that can be examined in relation to spending policies and their alignment with the real economy and the role of technology as an embodiment of knowledge in the resulting inter-causal relations. The Central Bank should have independence in setting its monetary goals and development of financial instruments for sustainable money and spending relations in an environment of technological change and the continuity of resource generation and its mobilization into the productive real economy.

The circular causation model that has been used in this paper can be well applied to its inherent system of structural econometric equations that can be used for studying interactions between monetary and fiscal policies (Gonzalez-Parano, 2010).

Objective

We will study the formalism underlying the nature of inter-causal relations in Figure 1. Such formalisms would be the appropriate ones if they can answer some specific questions, namely: is it possible to establish and sustain the relations of the mentioned complementarities in an age that looks up to the "wants" economy as opposed to the "needs" economy?; what are the contrary results to mainstream economic theory that emerge from the tripartite inter-causal relations?; what are the various policy-theoretic methods that can be utilized to solve the problem of sustainability as explained above and by Figure 1? These are questions that we will tackle in an ethical, social, and economic context in this paper. The model of inter-causality that will be used to study the possible complementary relations between the monetary, fiscal, and real economy variables is known as the circular causation model. It was originally used by Myrdal (1958). The same method was suggested by Desai (1989) in regards to circular causation modeling of endogenous money. This is the kind of monetary arrangement for establishing regimes of complementarities between money, spending, and the real economy. See further on in this paper.

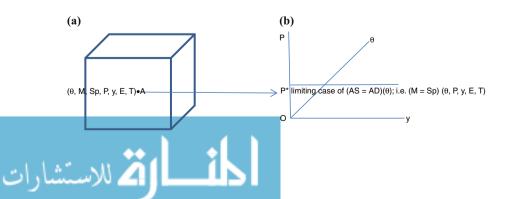


Figure 1.
Non-inflationary economic growth with complementary effects of (θ, M, Sp, P, y, E, T)

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Explanation

Let M denote the quantity of money; Sp the spending; P the price level; Y the real output; E the productive employment of factors; T the technological change e.g. weighted average of (output/factor) ratios as factor productivities. All these variables are induced by the knowledge variable, "\theta", "\theta" signifies the state of complementarities between the variables. It is treated as a knowledge variable (Wilson, 1998).

The learning nature of the inter-variable causality under the effect of θ -variable causes fuzzy neighborhoods around every point in Figure 1(a) and (b). The fuzzy configuration is not shown but is implied by the impact of " θ " all over. The point "A" explains the convergence of interrelations between the different sides of the Figure 1(a), all being induced by " θ " values. Hence taken together, the sides of the box interact and integrate at the point "A" with coordinate $\mathbf{x}(\theta) = \{\theta, M, Sp, P, y, E, T\}$. All the variables are thereby θ -induced. The result can be shown by Spatial Domain maps (the Appendix).

Due to the continuously differentiable properties of the interrelations between the variables caused by the continuity and differentiability of $\{\theta\}$ -values and their monotonic positive effects on the $\mathbf{x}(\theta)$ -vector and its functional transforms, we can write down the system of relations shown in the next section. Such a system is referred to as the circular causation system, because of the pervasively complementary relations between the variables caused by knowledge-induction.

The principal objective of the circular causation system is evaluation of the well-being function in both its: "estimation" form with actual data ("as is" state of the problem under study); and "simulation" form ("as it ought to be" state of the same problem under study). Quantitative policy analysis can follow thereafter (see the Appendix). These two steps together for quantitative policy and strategic evaluation comprise the meaning of "evaluation."

The circular causation model

Evaluate
$$W(\theta) = W(\mathbf{x}(\theta)), \quad \mathbf{x}(\theta) = \{x_1(\theta), x_2(\theta), \dots, x_n(\theta)\}$$
 (1)

Subject to,
$$x_i(\theta) = f_i(\theta, x_j(\theta))^p$$
, (2)

i, j = 1, 2, ..., n; $i \neq j$; p = 1, 2, ..., denote processes of evaluation of " θ ".

A specific form of the well-being function, $W(\theta)$, is "natural log-linearized" " θ "-function, as a continuous function of " θ " evolving over learning processes in concert with the induced $\mathbf{x}(\theta)$ -vector:

$$dW(\theta)/d\theta = \sum_{\mathbf{x}(\theta)} \left[\partial W^{\wedge}(\mathbf{x}(\theta)^{\wedge}) / \partial \mathbf{x}(\theta)^{\wedge} \right] \cdot \left(d\mathbf{x}(\theta)^{\wedge} / d\theta \right) > 0 \quad \text{identically,} \quad (3)$$

implying evolutionary learning of the estimators and the simulated well-being criterion function across processes, p = 1, 2, The simulated values and estimators are symbolized by ($^{\land}$).

The Systems (1)-(3) together comprise the circular causation system with its characteristics of interaction leading to integration and onwards to dynamic evolutionary learning processes.

In reference to the vector $(\theta, M, Sp, P, y, E, T)$ with the sequenced $x_i(\theta)$ -variables the following explanation of the circular causation interrelations of (1) and (2) can be made

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in respect of Figure 1. The need for positive interaction and integration between "M" and "Sp" is the important message that Blaug (1993) leaves in his study on economic methodology. The result is a perfectly elastic way of mobilizing money into volume of spending with the focus being on the exchange mechanism in the good and productive things of life, as can be evinced by transformation into a life-fulfilling regime of socioeconomic development. Such a regime would be characterized by stable prices and expansion of real output and employment with the continuous induction by appropriate technology. Thereby, life-sustaining regime of development is wholly characterized by continuously advancing complementary relationship between the vectors of variables as shown. Such an order of complementarities between the variables is caused epistemologically by the induction of unity of knowledge denoted by $\theta \in (\Omega, S)$, with Ω being the primal ontology of unity of knowledge is referred to as consilience in the foundational epistemological text. "S" denotes the functional ontology that maps the primal ontology into the world-system taken in its generality. The worldsystem example of our study is the particular case of money, spending, and real economy interrelationship with their inner dynamics of complementary change, "\theta" so defined in our particular study in world-system is the complementing guidance of unity of knowledge derived from the epistemological core and inducing the variables representing the good things of life. The choices are embodied in the greater purpose and objective of the shari'ah (magasid as-shari'ah) (Choudhury, 2011).

Circular causation method in the literature of socioeconomics

Indeed, the circular causation method has been used by important social scientists. Among them is Myrdal (1958). See Toner (1999a, b) on this topic. Pasinetti (1990) wrote on Piero Sraffa's contribution to circularity of the process of producing commodities by means of commodities. It is interesting to note how Pasinetti's remarks on this topic: "All commodities are, directly or indirectly, necessary to the production of all other commodities, and all of them return, at the end of each year, to the production process, either as necessary technical requirements or as necessary means of subsistence for the labourers [...]" (p. 230).

Circular causation idea has appeared in science as well (Hawking and Mlodinow, 2010). They write: "[...] for a general system, the probability of any observation is constructed from all the possible histories that could have led to that observation. Because of that (t)his method is called the 'sum over histories' or alternative histories' formulation of quantum physics" (p. 80). The aggregation of such histories is complex formulation (Kupka and Peixoto, 1993) as in the case of the circular causation relations, which are complex and non-linear because of the θ -induction of the coefficients as well as the $\mathbf{x}(\theta)$ -vector.

Likewise, Hawking and Mlodinow continue on: "Feynman showed that, for a general system, the probability of any observation is constructed from all the possible histories that could have led to that observation. Because of that his method is called the "sum over histories" or "alternative histories" formulation of quantum physics" (p. 80).

In sociology we have an equal explanation of circular causation by Fitzpatrick (2003, p. 128): "everything is a reproduction of other reproductions. Society explodes in on itself and we cannot liberate ourselves from the simulacra [...]."

Circular causation as philosophical method has found place in religious theory as well. On this topic Torrell (2005, pp. 27-36) writes: "The work (Summa) is in fact constructed according to a circular plan that draws the reader into the "going-out-from-returning-to" (exitus-reditus) movement, which is that of the entire universe coming from God to creation and returning to him as its final end."



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The money, spending, and real economy relationship between the Central Bank and commercial bank

Money ought to be fully mobilized into spending to cause the complementarities of the $\mathbf{x}(\theta)$ -vector and thereby the actualization of the simulated valuation of the well-being function. Consequently, there are three possible scenarios.

First, the deposits in the commercial bank are fully mobilized. This causes the (finance/deposit)-ratio to equal 1. Consequently, money ceases to be an interest-bearing asset in the absence of "bank-saving." Only returns on real assets matter and these replace interest rates by real rates of return. This possibility occurs when M=S (see Figure 1(b)), and which is possible when p* remains stable in Figure 1, causing thus a perfectly elastic expansion in "y" and "E"; and the endogenous role of technology. There is no statutory reserve in the Central Bank. This was also an Austrian economic idea (Yeager, 1997).

Second, when deposits are less than the financing needs, then the commercial banks together ask the Central Bank to create a quantity of money to meet the financing gap. The cost of producing this quantity of money is the seigniorage. This cost is recuperated from the commercial bank that in turn raises it from customers in service cost. The full mobilization of money for spending in real economy effects remains intact as in the above-mentioned case. The economic consequences of money, spending, and real economy in the context of complementarities between the $\mathbf{x}(\theta)$ -variables remain intact.

Third, when deposits exceed financing, creating surplus of deposits, the commercial bank deposits the excess with the Central Bank. This leaves full mobilization of funds into financing intact as above. The rest of the money-spending-real economy complementary relations remain the same. The excess in deposits with the Central Bank is secured by a proportionate amount of gold to protect currency values in deposit.

In each of the above three cases, there being no excess reserve with the commercial bank, a continuously creative evolution of non-inflationary output is maintained. Interest rate arbitrage is logically replaced by real rates of return raised from exchange mechanism. The resulting market economy in such cases makes the economy a most productive one.

Most significantly, it is noted that there is no fractional reserve requirement system in this case. Commercial banks hold all the deposits to be mobilized. Consequently, $R_{CB}/D = 0$, except in the third case. In this case too, holding savings with the Central Bank is like contractionary monetary policy. Now:

$$R_{CB}/D = \alpha; \quad R_{CB} = \alpha.D = P.G$$
 (4)

Here, R_{CB} denotes Central Bank reserve. α denotes a proportion of reserve with the Central Bank. P denotes price of gold. G denotes the quantity of gold required to protect the exchange value of R_{CB} .

In each of the above cases we note that the quantity of gold required for protecting the currency exchange value of R_{CB} is inversely related to the mobilization of saving deposits into the real economy. The more is the activity of money, spending, and the real economy the less is the amount of gold required to protect the currency value, and vice versa.

The currency, gold, and monetary reserve relationship

Let "e" denote the exchange value of currency in circulation by spending.



C denote the amount of internationally traded currency in circulation. Thus, (1-C) are domestically used. The less is traded abroad as domestic economic expansion advances:

$$e = (C/\$) = C(\theta, M, Sp, y, p, E, T)/\$ (\downarrow)$$
(5)

As domestic complementarities in $\mathbf{x}(\theta)$ -vector increases. Thus, "e" strengthens in favor of the trading country.

The gold relationship of expression (5) is the following one:

$$e = (C/\$) = C(\theta, M/G, Sp, y, p, E, T)/\$(\downarrow)$$
 (6)

Let m denote (M/G); p denotes price level:

$$dm/d\theta = m[g_{M,\theta} - g_{G,\theta}]$$
 (7)

< 0 when there is higher rate of monetary growth in circulation, and thus lower rate of growth of gold stock in respect of " θ " effect. < 0 when there is lower rate of monetary growth in circulation, and thus higher higher rate of growth of gold stock in respect of " θ ".

Now, the value for:

$$de/d\theta = (other positively related terms) + (\partial C/\partial m).(dm/d\theta)$$
 (8)

moves with in a narrow band even as the monetary R_{CB} moves narrowly around zero value.

The preceding explanation regarding monetary arrangements in a 100 per cent Reserve Requirement Monetary System (100%RRMS) in the heterodox ethico-economic financial economy system of pervasive complementarities between the $\mathbf{x}(\theta)$ -variables by virtue of its epistemology of unity of knowledge (Choudhury, 1997) is quite different from the explanation in the fractional reserve requirement monetary system. In the 100%RRMS the more money is available and mobilized into the real economy with its inner complementary dynamics the happier is the economy by its social well-being index (Haines, 1987).

Furthermore, the stability of the exchange rate within a narrow band with the gold standard playing its stabilizing role in the money, spending, and real economy circular causation relations with the complementarities of the rest of the variables induced by "0," proves the following result: a small amount of gold stock supporting saving in the CB-monetary reserve causes a large mobilization of financing to support elastic activities in the real economy. Besides, the non-inflationary economic growth is maintained under the effect of stable exchange rate without the interest rate-exchange rate intervention.

We also note that, in the structure of the life-fulfillment development regime within which the money, spending, real economy, and other supporting activities sustain their complementarities, the role of the market economy is heightened over unnecessary government intervention. The strategies and policies enacted to guide the economy, and the technological change are all endogenous in nature. Money becomes endogenous in nature (Desai, 1989; Choudhury, 1997). The market oriented role of the economy changes the structure of relationship between the Central Bank, commercial banks, and the real economy ensemble. The effective endogenous relationships

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between the complementary variables ensemble generate increasing returns to scale in the simulated well-being function (Romer, 1986).

According to the characteristic of pervasive complementarities between the strategies and payoffs, each of these elements together comprising a process (p=1,2,3,4,5,6 mutually exclusive processes in the matrix of Table I) in the matrix as explained above in respect of the vector $\mathbf{x}(\theta)$. If we further extend such complementary relations to inter-processes, then the game-theoretic method of systemic social evaluations breaks down. That is because game-theoretic strategies and payoffs denote pursuits of conflicting interests of participants. Hence the processes as various systemic strategies and payoffs reflect such conflicting interests and remain individuated.

Circular causation and knowledge-induced game-theoretic strategies and payoffs: comparison and contrast

We now proceed along these lines to compare and contrast game-theoretic method of evaluation of the Figure 1 with the method of circular causation.

Each of the rows and columns of the matrix in Table I denotes a mutually exclusive process. From our previous explanation regarding the complementary characteristic of $\mathbf{x}(\theta)$ caused and regenerated by the epistemic influence of $\theta \in (\Omega, S)$, we note that each of the payoffs across the two-player strategies is a function of the extended vector. This more comprehensive vector $\mathbf{x}(\theta) = \{\theta, C, m, Sp, y, p, e, E, T\}(\theta)$ includes all the complementary variables to establish the result shown in Figure 1. Each of the ensuing process of payoffs across strategies is "evaluated" in the sense of circular causation relations between the extended variables for the objective of evaluating the well-being function in terms of complementarities between the variables by the induction of " θ ."

In the end, with each evaluated well-being by process being denoted by W_i , i = 1, 2, 3, 4, 5, 6; the collective well-being ensemble of the aggregated processes is denoted by the following expression:

$$W(\theta) = \Pi_i W_i(\theta)_i^{\alpha} \tag{9}$$

Each well-being index $W_i(\theta)$ is similarly defined and detailed as in expression (1) and onwards. α_I denotes the elasticity of $W_i(\theta)$ to $W(\theta)$. The technical implication of mutual independence between the processes as the property emanating from the gametheoretic formulation is carried through in such a formulation of well-being. Consequently, in the study of inter-process causality between the defining variables of payoffs across strategies according to processes the circular causation method proves to be more versatile. Similarities between game-theoretic and circular causation approach do remain.

Table I.
Knowledge-induced
game-theoretic
strategies and
payoffs

	Central Bank			
Commercial Bank	1	2	3	
1 2 3	(R = 0, D/F = 1) (R = 0, D/F > 1) (R = 0, (D/F) < 1)	(R = 0, (D/F) = 1) (R = 0, (D/F) > 1) (R = 0, (D/F) < 1)	$(R = \alpha D, (D/F) = 1)$ $(R = \alpha D, (D/F) > 1)$ $(R = \alpha D, (D/F) < 1)$	

Game-theoretic modeling of 100% RRMS in learning space of strategies and payoff



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Conclusion

This exploratory paper has established the theme of money, spending, and real economy complementary relationships in the framework of the circularly causal organic relations between these variables and their inner details. The methods that emanate at the modeling and empirical levels are found to be of profound epistemological nature. The inherent methodological issues involve formulation of circular causation relations as was explained. Besides, while involving the interrelations between the Central Bank and the conventional bank in terms of monetary, fiscal, and real economy interaction a gametheoretic model was referred to. In the end, the formalism involving the complementary relations between money, fiscalism, market spending, and the real economy was shown to be rich in invoking relevant policies. One such policy referred to is the treatment of the gold standard replacing the regime of paper money and interest rates.

While the paper was of an exploratory nature, its further implications point out a substantive avenue of conceptualization, modeling, economic structuring, and monetary and real economy policies that promote socioeconomic development by means of such complementary relations. The end result would be the realization of stabilization and economic expansion. While such a possibility involves both endogenous (circular causation) organic relations between relevant variables, it is also a function of endogenous technological change.

In Desai's model of endogenous monetary dynamics the symbols of the endogenous variables can be taken as, x_1 : money; x_2 : spending; x_3 : real economy variables; x_4 : technological change. The nature of such endogenous organic inter-variable and intercausal relations conveys the intrinsic role of knowledge and learning in terms of the circular causation relations. Such evolutionary learning as the epistemological element in the circular causation relations represents the nature of systemic or endogenous ethics. Finally, the whole system of all such variables defining monetary and spending regimes now turns out to be of an endogenous type. The implications are thereby vast.

The exploratory methodology along with its methodical formalism now form an original contribution in the field of endogenous money, spending, and real economy relational regimes with all the details. Such an emergent inter-causal system of circular causation relations between the variables can now be written as an extension of Desai's model of endogenous money.

The vector of inter-causal variables is $\{x_1, x_2, x_3, x_4, \theta\}$, with the circular causation relations explicating the ethics of evolutionary learning by inter-variable causality. Such a system of circular causation relations is denoted by:

$$\begin{split} x_1 &= f_1(x_2,x_3,x_4)[\theta]; \quad x_2 = f_2(x_1,x_3,x_4)[\theta]; \quad x_3 = f_3(x_1,x_2,x_3)[\theta]; \\ x_4 &= f_4(x_1,x_2,x_3)[\theta]; \quad \theta = F(x_1,x_2,x_3,x_4). \end{split}$$

The last of these relations measures the ethical nature of knowledge emanating from the inter-causal complementary nature and the degree to which this exists and is improvable between the endogenously related variables.

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Further reading

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Appendix. Spatial domain analysis (SDA) representations along with circular causation estimation results

The Appendix here explains the results of evaluating the circular causation modeled system of equations in their structural form. The term "evaluation" here means first, "estimation" of the equations of circular causation. Second, "estimation" is followed by "simulation" of the "estimated" results so as to generate better degrees of complementarities between the monetary, fiscal, and real economy variables. In the simulation stage of the empirical exercise here SDA method is used. This method within the Geographical Information System (GIS) now adapted to study socioeconomic topography can automatically generate a sea of possible coefficients around a suggested value of each coefficient simulation. Such various simulated coefficient values provide multi-possibilities of choices on the theme of integrating the monetary, fiscal, and real economy variables into complementary forms. Every simulated coefficient values of complementarities is linked with important strategic and policy implications on the theme of integrated complementarities between monetary, fiscal, and real economy regimes of changes. This was the principal theme of this paper. We proceed as follows.

Let M denotes quantity of money; IN denotes spending (investment); Trade as shown.

 θ are given ordinal values proportionately to the averaged degrees of complementarities between the variables after ranking such values by individual variable columns:

$$\ln\theta = -0.514 + 0.792 \ln M - 0.244 \ln IN + 0.123 \ln TRADE \tag{A1}$$

 $S = 0.0348958 R^2 = 97.2\% R^2 \text{ (adj)} = 96.6\%$, Durbin-Watson statistic = 1.87454

$$lnTRADE = 6.53 - 1.63 lnM - 0.022 lnIN + 1.57 ln\theta$$
 (A2)

 $S = 0.124388 R^2 = 87.1 \% R^2 \text{ (adj)} = 84.5 \%$, Durbin-Watson statistic = 0.887537

$$\ln M = 1.20 + 0.244 \ln IN - 0.176 \ln TRADE + 1.09 \ln \theta \tag{A3}$$

 $S = 0.0408542 R^2 = 88.1\% R^2$ (adj) = 85.8%, Durbin-Watson statistic = 1.57629

$$lnIN = 1.32 + 2.27 lnM - 0.022 lnTRADE - 3.11 ln\theta$$
 (A4)

 $S = 0.124582 R^2 = 96.0\% R^2 \text{ (adj)} = 95.1\%$, Durbin-Watson statistic = 1.45823

Simulated equations (using the SDA numerical choices for selected coefficients):

$$lnTRADE = 6.53 - 1.417.lnM - 0.022 lnIN + 1.57 ln\theta$$
 (A5)

$$\ln M = 1.20 + 0.103 \ln IN - 0.103 \ln TRADE + 1.09 \ln \theta \tag{A6}$$

$$lnIN = 1.32 + 1.672 lnM - 0.022 lnTRADE - 3.11 ln\theta$$
 (A7)

$$\ln\theta = -0.514 + 0.792 \ln M - 0.244 \ln IN + 0.123 \ln TRADE$$
 (A8)

The OLS estimation results using Minitab software.



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Regression analysis

Regression analysis: lnM×100, lnIN%, lnTRADE×100, lnθ:

(1) Regression analysis: lnM vs lnIN, lnTRADE, lnθ.

The regression equation is:

$$\ln M = 1.20 + 0.244 \ln IN - 0.176 \ln TRADE + 1.09 \ln \theta$$
 (A9)

Predictor	Coef	SE Coef	T	P
Constant	1.2043	0.5362	2.25	0.040
lnIN	0.2444	0.0564	4.33	0.001
lnTRADE	-0.1763	0.0715	-2.46	0.026
lnθ	1.0860	0.1129	9.62	0.000

 $S = 0.0408542 R^2 = 88.1\% R^2$ (adj) = 85.8%, Durbin-Watson statistic = 1.57629

(2) Regression analysis: lnIN vs lnM, lnTRADE, lnθ.

The regression equation is:

$$lnIN = 1.32 + 2.27 lnM - 0.022 lnTRADE - 3.11 ln\theta$$
 (A10)

Predictor	Coef	SE Coef	T	P
Constant	1.317	1.859	0.71	0.490
lnM	2,272	0.525	4.33	0.001
lnTRADE	-0.022	0.259	-0.08	0.935
lnθ	-3.106	0.454	-6.84	0.000

 $S = 0.124582 R^2 = 96.0\% R^2 \text{ (adj)} = 95.1\%$, Durbin-Watson statistic = 1.45823

(3) Regression analysis: lnTRADE vs lnM, lnIN, lnθ.

The regression equation is:

$$lnTRADE = 6.53 - 1.63 lnM - 0.022 lnIN + 1.57 ln\theta$$
 (A11)

Predictor	Coef	SE	T	P
Constant	6.532	0.8467	7.71	0.000
lnM	-1.635	0.6632	-2.46	0.026
lnIN	-0.022	0.2577	-0.08	0.935
lnθ	1.568	0.8266	1.90	0.077

 $S = 0.124388 R^2 = 87.1\% R^2 \text{ (adj)} = 84.5\%$, Durbin-Watson statistic = 0.887537

(4) Regression analysis: lnθ vs lnM, lnIN, lnTRADE.

The regression equation is:

$$\ln\theta = -0.514 + 0.792 \ln M - 0.244 \ln IN + 0.123 \ln TRADE$$
 (A12)

Predictor	Coef	SE	T	P
Constant	-0.5142	0.5125	-1.00	0.332
lnM	0.7923	0.0824	9.62	0.000
lnIN	-0.2437	0.0357	-6.84	0.000
lnTRADE	0.1234	0.0651	1.90	0.077

 $S = 0.0348958 R^2 = 97.2\% R^2 \text{ (adj)} = 96.6\%$, Durbin-Watson statistic = 1.87454

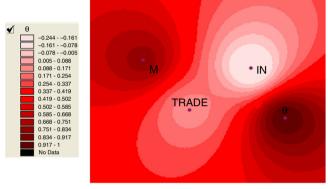
SDA results

Spatial Domain Analysis (SDA) is a methodology within GIS that maps the interrelations between variables (hence entities), which can be represented in the real space. But when we treat the socioeconomic and abstract socio-scientific cases by means of SDA, we take the real space to be represented by measured variables. Such measurements can be actual data or ordinalized representations.

Note that the measured (quantitative version) social well-being as an ethical index remains almost unchanged in the estimated and the normative equations because of the correct signs of the variables related to lnθ. But predictor (simulated) values are different from the estimated values.

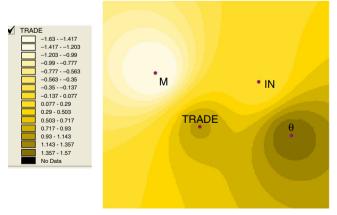
Quantitative modeling of mathematical relationships

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Note: $\ln\theta = -0.514 + 0.792 \ln M - 0.244 \ln IN + 0.123 \ln TRADE$

Figure A1. SDA Analysis in lnθ vs InM, InIN, InTRADE



Note: $lnTRADE = 6.53 - 1.63 lnM - 0.022 lnIN + 1.57 ln\theta$ simulated coefficient value (-1.417)

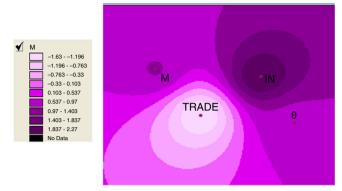
Figure A2. SDA Analysis InTRADE vs InM, InIN, Inθ



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Figure A3. SDA Analysis InM vs ININ, InTRADE, Inθ



Note: $lnM = 1.20 + 0.244 \ lnIN - 0.176 \ lnTRADE + 1.09 \ ln\theta$ simulated coefficient value (0.103) (0.103)



Figure A4. SDA Analysis InIN vs InM, InTRADE, Inθ

Note: $lnIN = 1.32 + 2.27 lnM - 0.022 lnTRADE - 3.11 ln\theta$ simulated coefficient value (1.672)

Corresponding author

Professor Masudul Alam Choudhury can be contacted at: masudc60@yahoo.ca

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